Switching of discrete optical solitons in engineered waveguide arrays

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We demonstrate a simple concept for controlling nonlinear switching of discrete solitons in arrays of weakly coupled optical waveguides, for both cubic and quadratic nonlinear response. Based on the effective discrete nonlinear equations describing light propagation in the waveguide arrays in the tight-binding approximation, we demonstrate the key ideas of the array engineering by means of a steplike variation of the waveguide coupling. We demonstrate the digitized switching of a narrow input beam for up to 11 neighboring waveguides, in the case of the cubic nonlinearity, and up to 10 waveguides, in the case of the quadratic nonlinearity. We discuss our predictions in terms of the physics of the engineered Peierls-Nabarro (PN) potential experienced by strongly localized nonlinear modes in a lattice, and calculate the PN potential for the quadratic nonlinear array for the first time.

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I. INTRODUCTION

Discrete nonlinear systems are known to support self-localized modes that exist due to an interplay between a coupling between the lattice sites and nonlinearity [1]. Such spatially localized modes of discrete nonlinear lattices existing without defects are known as discrete solitons or intrinsic localized modes; they appear in many diverse areas of physics such as biophysics, nonlinear optics, and solid state physics [2,3]. More recently, such modes have been predicted in the studies of the Bose-Einstein condensates in optical lattices [4] and photonic-crystal waveguides and circuits [5].

One of the most important applications of discrete solitons is found in nonlinear optics where discrete optical solitons were first suggested theoretically by Christodoulides and Joseph [6] for an array of weakly coupled optical waveguides. Because the use of discrete solitons promises an efficient way to realize and control multiport nonlinear switching in systems of many coupled waveguides, this field has been explored extensively during the last 10 years in a number of theoretical papers (see, e.g., Refs. [7–9], as an example). More importantly, discrete solitons have also been generated experimentally in fabricated periodic waveguide structures (see, e.g., [10,11] for original papers reporting on the experimental observations and also [12–14] for recent review papers).

The majority of theoretical studies conducted so far is devoted to the analysis of different types of stationary localized modes in discrete nonlinear models and their stability. Consequently, experimental papers have reported on the observation of self-trapped states in the periodic systems with broken translational symmetry and some of their properties, in both focusing and defocusing regimes [12–14]. However, only very few studies and experimental demonstrations addressed more specific properties of localized modes introduced by discreteness such as the soliton steering in and discreteness-induced trapping by the lattice (see, e.g., Ref. [15]). As a result, very little effort has been made so far to link these findings with realistic applications of discrete solitons for multiport all-optical switching.

Indeed, one of the major problems for achieving controllable multiport all-optical switching of discrete solitons in waveguide arrays is the existence of an effective periodic Peierls-Nabarro (PN) potential which appears due to the lattice discreteness. As a consequence of this potential, a narrow large-amplitude discrete soliton does not propagate freely in the lattice and, instead, becomes trapped by the array. Several ideas to exploit the discreteness properties of the array for digitized all-optical switching have been suggested [16,17]. However, the main result of those earlier studies is the observation that the discrete solitons can be well controlled only in the limit of broad beams, whereas the soliton dynamics in highly discrete arrays has been shown to be more complicated and even chaotic [17].

In this paper, we explore in detail an effective way to control nonlinear switching of discrete solitons in arrays of weakly coupled optical waveguides earlier suggested in our brief letter [18]. First, using the discrete model valid in the tight-binding approximation, we estimate the PN potential experienced by a strongly localized nonlinear mode that is kicked initially in a cubic nonlinear waveguide array. The result suggests a possible control mechanism for the switching of strongly localized excitations by means of a steplike variation of the waveguide coupling. For particular types of the engineered arrays, we are able to demonstrate the digitized switching of a narrow input beam for up to 11 waveguides, for the case of cubic nonlinearity. We also extend the concept of controllable digitized switching of discrete optical solitons to the case of quadratic nonlinear waveguide arrays, where the experimental observation of discrete optical solitons has been reported very recently [19]. Here, we obtain, for the first time to our knowledge, the PN potential for the discrete soliton and demonstrate numerically the digitized switching for up to 10 waveguides.

The paper is organized as follows. In Sec. II we study the arrays of cubic nonlinear waveguides. First, we consider the system dynamics described by the discrete nonlinear Schrödinger equation, and show how to modulate the waveguide coupling in order to suppress the chaotic dynamics and

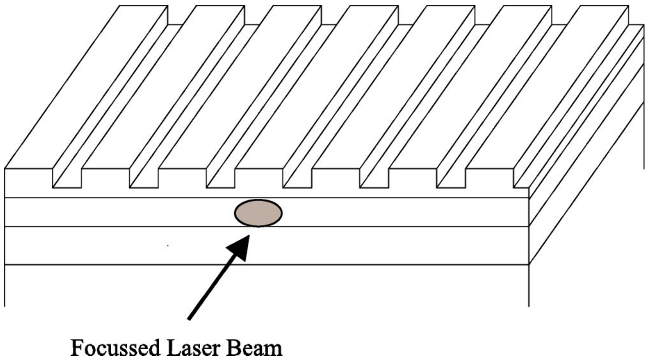

FIG. 1. Example of a homogeneous waveguide array and the generation of a discrete soliton by exciting a single waveguide.

achieve fully controllable digitized switching. Next, in Sec. III we extend our analysis to the arrays of weakly coupled quadratic nonlinear waveguides, where discrete quadratic solitons are composed of the coupled beams of the fundamental and second-harmonic fields. Finally, Sec. IV concludes the paper.

II. CUBIC NONLINEAR WAVEGUIDES

The most common theoretical approach to studying the discrete optical solitons in arrays of weakly coupled optical waveguides is based on the decomposition of the electric field of the periodic photonic structure into a sum of weakly coupled fundamental modes excited in each waveguide of the array; in solid-state physics this approach is known as the tight-binding approximation. According to this approach, the wave dynamics is described by an effective discrete nonlinear Schrödinger (DNLS) equation that possesses spatially localized stationary solutions in the form of discrete localized modes. Many properties of the discrete optical solitons can be analyzed in the framework of this approach and the DNLS equation [6,9].

Homogeneous arrays. A standard model of a weakly coupled array of cubic nonlinear waveguides is described by the DNLS equation [6] that we write in the dimensionless form (see, e.g., Ref. [9]),

$$i\frac{du_n}{dz} + V(u_{n+1} + u_{n-1}) + \gamma|u_n|^2 u_n = 0, \tag{1}$$

where u_n is the amplitude of the effective electric-field envelope of the fundamental mode excited in the nth waveguide (normalized to the square root of the peak power), V is proportional to the overlap integral of the electric field modes, and it characterizes the coupling between the neighboring waveguides (normalized to the inverse of the coupling length), and z is the propagation distance along the waveguide normalized to the coupling length. Parameter γ is the effective waveguide nonlinearity associated with the Kerr effect of the core material, normalized to the product of inverse peak power and inverse coupling length. Figure 1 shows a typical experimental structure of a quasi-one-dimensional homogeneous waveguide array and the excitation scheme for generating a discrete optical soliton.

Steering and trapping of discrete optical solitons have been analyzed in the framework of the model (1) in a number of theoretical studies. Being kicked by an external force, the discrete soliton propagates through the lattice for some distance, but then it gets trapped by the lattice due to the discreteness effects. For a stronger kick, the output soliton position fluctuates between two (or more) neighboring waveguides making the switching uncontrollable [17].

In order to show this feature, first we consider homogeneous arrays and select an input profile in the form of a narrow sechlike beam localized on a few waveguides following the earlier study by Królikowski *et al.* [20]:

$$u_n(0) = A\,\mathrm{sech}[A(n-n_c)/\sqrt{2}]e^{-ik(n-n_c)}, \tag{2}$$

for $n-n_c=0,\pm 1$, and $u_n(0)=0$, otherwise. For the particular results presented below, we consider an array of 101 waveguides and place the beam at the middle position, $n_c=50$. The maximum normalized propagation distance used in our simulations is $z_{\max}=45$. Parameter k in the ansatz (2) has the meaning of the transverse steering velocity of the beam, in analogy with the continuous approximation. It describes the value of an effective kick of the beam in the transversal direction at the input, in order to achieve the beam motion and shift into one of the neighboring (or other desired) waveguide outputs.

In our simulations, we control the numerical accuracy by monitoring the two conserved quantities of model (1), the soliton power

$$P = \sum_n |u_n(z)|^2, \tag{3}$$

and the system Hamiltonian,

$$H = -\sum_n \{V(u_n u_{n+1}^* + u_n^* u_{n+1}) + (\gamma/2)|u_n|^4\}. \tag{4}$$

The input condition (2) does not correspond to an exact stationary solution of the discrete equation (1) even for $k=0$ and, as the input kick ($k\neq 0$) forces the localized wave move to the right ($k<0$) or left ($k>0$), its motion is accompanied by some radiation. The effective lattice discreteness can be attributed to an effective periodic potential, the PN potential, which is dynamic and changes in time. Due to both, the strong radiation and the presence of the PN barrier which should be overtaken in order to move the beam transversally, the discrete soliton gets trapped at one of the waveguides in the array. In most of the cases, the shift of the beam position to the neighboring waveguide is easy to achieve, as shown in many studies [17]. However, the soliton switching becomes rather complicated and even chaotic. This is shown in Fig. 2 where, for a fixed value of the input angle, a slight variation in the beam intensity results in erratic switching of the beam. A complete sweep is shown in Fig. 3 (left-hand side).

Modulated arrays. In this paper, we suggest to modulate the coupling in the waveguide array in order to achieve a controllable output and to engineer the switching results. What this modulation of the couplings does is to affect the PN barrier, providing us with a simple physical mechanism

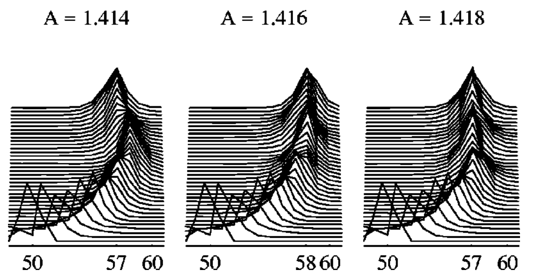

FIG. 2. An example of erratic switching of a localized input beam with a slight variation of the beam intensity in a homogeneous array, for $V=\gamma=1$.

for fine-tuning and control of the beam self-trapping.

To justify the validity of our concept, we perform a qualitative estimate of the PN barrier in the framework of the applicability of the discrete model and perturbation theory. We study the case of strongly localized modes [21] propagating in a homogeneous waveguide array with identical coupling between the neighboring waveguides, described by Eq. (1). We consider a general localized mode that we want to propagate throughout the array. Due to discreteness, our system lacks the translational invariance and, as a result, some external "kick" must be supplied in order to force the mode to move. Another way to look at this problem is to consider that, because of the lattice discreteness, the localized mode "sees" a potential barrier (the PN barrier), whose height depends on the effective discreteness of the system as seen by the excitation [21]. Thus, for wide modes, the barrier will be smaller that for narrow modes. A rough estimate of this PN barrier can be obtained by equating it to the difference in the values of the Hamiltonian, between the mode centered at a waveguide (odd mode) and the mode centered between two neighboring waveguides (even mode) [21].

In order to evaluate a change of the PN barrier for the mode initially kicked by an external force, we introduce an initial phase tilt that is proportional to the factor $\sim\exp(-ikn)$ in the discrete case. Our purpose is not only to provide an extension to earlier results [21], but also to study, for the first time to our knowledge, the variation of the effective PN potential for an initially kicked localized mode.

Odd modes. We consider a strongly localized mode (SLM) in the form of three excited sites,

$$u_n(z) = u_n e^{i\lambda_1 z} \approx u_0\{0\ldots,0,\epsilon_1 e^{ik},1,\epsilon_1 e^{-ik},0,\ldots,0\}e^{i\lambda_1 z}, \tag{5}$$

where u_0 is the mode amplitude, k is the parameter of the initial "kick" (an effective transverse angle) applied to the mode, λ_1 is the longitudinal propagation constant, and ϵ_1 is a small parameter, to be determined from Eq. (1). After substituting Eq. (5) into Eq. (1) and keeping only linear terms in ϵ_1, we obtain

$$\lambda_1 = 2\epsilon_1 V\cos(k) + \gamma u_0^2$$

and $\epsilon_1 = V\cos(k)/\lambda_1$, so that

$$\lambda_1 \approx \gamma u_0^2, \quad \epsilon_1 \approx V\cos(k)/\gamma u_0^2 \ll 1. \tag{6}$$

Even modes. In this case, the SLM mode has the form

$$\tilde{u}_n(z) = \tilde{u}_n e^{i\lambda_2 z} \approx \tilde{u}_0\{0\ldots,0,\epsilon_2 e^{ik},1,e^{-ik},\epsilon_2 e^{-2ik},0,\ldots,0\}e^{i\lambda_2 z}, \tag{7}$$

where, as above, $\tilde{u}_0$ is the amplitude of the even mode, k is the initial angle or effective parameter of the initial "kick," λ_2 is the longitudinal propagation constant of the even mode, and ϵ_2 is a small parameter. After substituting Eq. (7) into Eq. (1) and keeping only linear terms in ϵ_2, we obtain

$$\lambda_2 = (1+\epsilon_2)V\cos(k) + \gamma\tilde{u}_0^2$$

and $\epsilon_2 = V\cos(k)/\lambda_2$, so that

$$\lambda_2 \approx V\cos(k) + \gamma\tilde{u}_0^2, \quad \epsilon_2 \approx \frac{V\cos(k)}{V\cos(k)+\gamma\tilde{u}_0^2} \ll 1. \tag{8}$$

From Eqs. (6) and (8) we come to the conclusion that, in order to have strongly localized modes, the nonlinear contribution described by the term γu_0^2(or $\gamma\tilde{u}_0^2$) must be much larger than the linear term described by the term $V\cos(k)$. Now, for calculating the PN barrier, we should relate the amplitudes of the modes of two different symmetries. One way is to think of both the modes as different states of a single effective mode shifted by a half lattice site along the

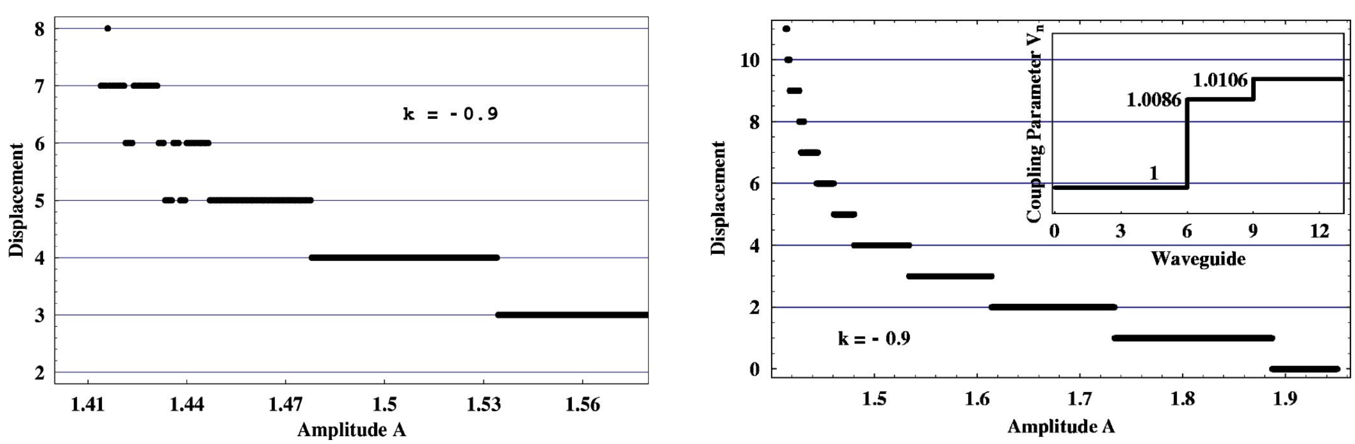

FIG. 3. Digitized switching of a discrete soliton in a cubic nonlinear waveguide array by varying the beam intensity, for $k=-0.9$ and $\gamma=1$. Left, homogeneous array for $V=1$. Right, engineered array for the coupling modulation shown in the inset.

chain. This means that the power content of both modes must be identical, since the power $P=\Sigma_n|u_n(z)|^2$ is a conserved quantity. To the first order in ϵ_1 and ϵ_2, we obtain

$$P_{\text{odd}} = u_0^2 + O(\epsilon_1^2), \quad P_{\text{even}} = 2\tilde{u}_0^2 + O(\epsilon_2^2). \tag{9}$$

Thus, the relation $P_{\text{odd}}=P_{\text{even}}$ implies $u_0^2 \approx 2\tilde{u}_0^2$. We are now in position to compute H_{odd} and H_{even} for a strongly localized mode, using the above relation and Eqs. (4), (6), and (8),

$$H_{\text{odd}} \approx -\frac{\gamma}{2}u_0^4 + O(\epsilon_1^2),$$
$$H_{\text{even}} \approx -\frac{\gamma}{4}u_0^4 - u_0^2 V\cos(k) + O(\epsilon_1 \cdot \epsilon_2), \tag{10}$$

which implies that the PN barrier $\Delta^{(3)}$ for the nonlinear cubic array is given by

$$\Delta^{(3)} = H_{\text{odd}} - H_{\text{even}} \approx -\frac{\gamma}{4}u_0^4 + u_0^2 V\cos(k). \tag{11}$$

In comparison with the previously obtained result for the PN barrier [21], Eq. (11) adds an extra, albeit small, term that shows how the PN barrier is modified for the mode initially kicked in the lattice. Indeed, in addition to the first term dependent on the mode amplitude, Eq. (11) also includes a linear term proportional to the factor $V\cos(k)$, whose magnitude could be modified by a judicious adjustment of the waveguide couplings and/or the value of the initial kick.

Dependence of the PN barrier on the mode coupling suggests that, if we wish to find a way to engineer the value of the PN barrier in the lattice, we should study the properties of a modified model described by the evolution equation

$$i\frac{du_n}{dz} + V_{n+1,n}u_{n+1} + V_{n,n-1}u_{n-1} + \gamma|u_n|^2 u_n = 0, \tag{12}$$

where the parameter $V_{n,m}$, describing the coupling between two guides with the indices n and m, can be modified through a change in the spacing between the waveguides. To study the beam steering in this model, we again use as an initial condition the sechlike profile (2), although this is not a really fundamental limitation, as argued below.

We mention that a variation of the waveguide coupling in the array constitutes the starting point for our concept of the waveguide array engineering. A change of the couplings breaks the symmetry between the beam motion to the right and left at the moment of trapping, thus eliminating chaotic trapping observed in the case of homogeneous arrays.

We have tested different types of modulation in the array coupling and the corresponding structures of the waveguide superlattices. The procedure is as follows: For a homogeneous array (when V is constant), we launch a transversally localized beam of amplitude A, with the transversal momentum k, and record the position of the output beam after many coupling lengths, as a function of the initial amplitude A. This switching plot has a ladderlike structure, as shown in Fig. 3 (left-hand side). However, the output beam position jumps discontinuously at certain amplitudes A. Thus, the ladder (switching curve) is not optimal (i.e., not monotonic). In order to apply the concept of the coupling engineering at each of these bad output beam positions, we perform a slight change in their coupling to their neighbors, until that portion of the ladder is restored to a monotonic form. At the end of this process, one obtains the optimal coupling profile for the given k that allows switching to a certain number of sites [see Fig. 3 (right-hand side)]. It should be stressed that $V_{n,m}$, the optimal coupling profile, will depend on momentum k (angle). One can also keep the amplitude A fixed and vary the kick k, and proceed as before, modulating the coupling in order to obtain a monotonic "ladder" for switching. In this case, the $V_{n,m}$ will depend on the amplitude A.

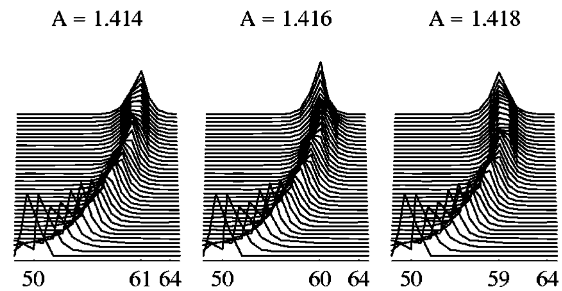

FIG. 4. Same as in Fig. 2, but in the engineered waveguide array with the coupling modulation shown in the inset of Fig. 3.

An example of one such optimized structure, where we modulate the coupling parameter $V_{n,m}$ in a steplike manner, is shown in the inset of Fig. 3. This also shows the discrete position of the soliton at the output as a function of the amplitude of the input beam, at a fixed value of the steering parameter $k=-0.9$. In a remarkable contrast with other studies (see, e.g., Ref. [17]), the coupling modulation allows the achievement of a controllable digitized switching of discrete optical solitons in the array with very little or no distortion.

As is shown in Fig. 4, by decreasing the amplitude of the input pulse at a fixed value of the steering parameter k (in our example fixed to be $k=-0.9$), it is possible to achieve self-trapping of the discrete soliton by the lattice at some (short) distance from the input at different waveguide positions. Due to the steplike modulated coupling, we create a selection between the beam motion to the right and left at the moment of trapping thus suppressing or eliminating the chaotic trapping observed in homogeneous waveguide arrays. In this way, we achieve a controllable digitized nonlinear switching where the continuous change of the amplitude of the input beam results in a quantized systematic displacement of the output beam by an integer number of waveguides. Consequently, for the parameters discussed above we observe almost undistorted switching up to 11 waveguides. Incidentally, we notice here that the use of a linear ramp potential (e.g., in the form $V_n=an$) for this purpose does not lead to an effective switching. Instead, it makes the soliton switching even more chaotic due to the phenomenon of Bloch oscillations which become randomized in the nonlinear regime [22].

In Fig. 5 we show another example of an homogeneous array (left-hand side) and an array with the optimized coupling modulation (right-hand side), this time as a function of the effective input kick, for a fixed beam intensity. In this case, we can achieve completely controlled switching up to nine waveguides.

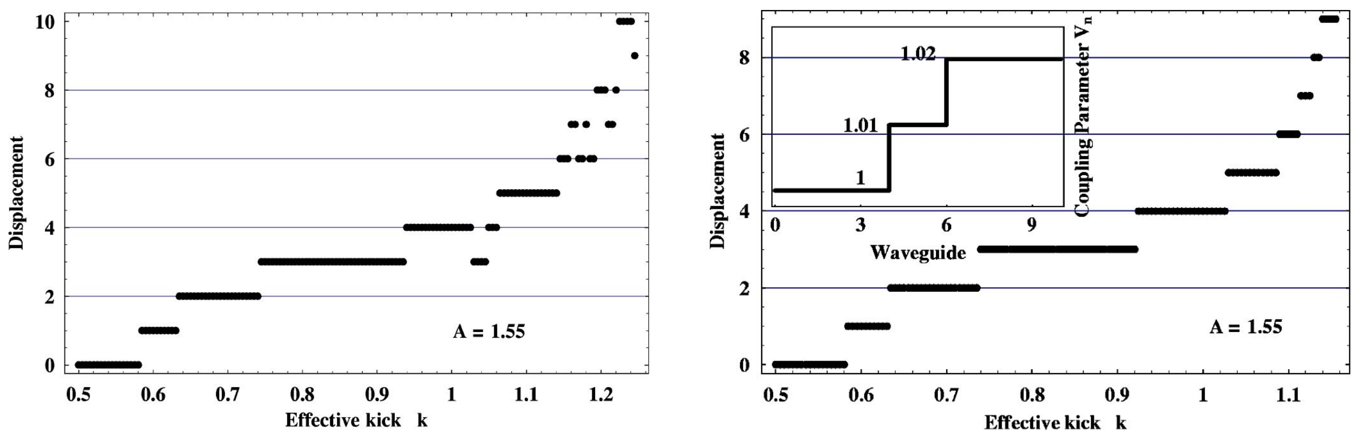

FIG. 5. Digitized switching of a discrete soliton in a cubic nonlinear waveguide array by varying the beam input angle, for $A=1.55$ and $\gamma=1$. Left, homogeneous array for $V=1$. Right, engineered array for the coupling modulation shown in the inset.

If the input beam was to excite initially five waveguides instead of three creating in this manner a wider excitation (i.e., being closer to the continuum limit), one could expect a smaller amount of radiation emitted. However, this would imply a longer distance before the beam gets trapped by one of the waveguides in the array due to the effective PN potential. Also, this means that one could, in principle, switch the soliton beam to any desired waveguide in the waveguide array, no matter how far; it would just be a matter of choosing an initial beam wide enough, i.e., closer to the continuum (in addition to optimize the coupling in a stepwise manner), by removing the random selection between the directions and suppressing the beam random switching.

Another observation is that the sechlike initial profile is not really fundamental. We have verified that similar dynamics is observed for other types of input beam profiles, including a kicked nonlinear impuritylike input of the form [23]

$$u_n(0) = A(P)\left(\frac{P - A(P)^2}{P + A(P)^2}\right)^{|n-n_c|/2} e^{-ik(n-n_c)}. \qquad (13)$$

where $A(P)^2=\sqrt{P^2-(2V/\gamma)^2}$ and $(P>2V/\gamma)$. The reason for this universal behavior seems to rest on the observation that for any system with local nonlinearity a narrow initial profile will render the system into an effective linear one containing a small nonlinear cluster (or even a single site); the bound state will therefore strongly resemble that corresponding to a nonlinear impurity [24].

III. QUADRATIC NONLINEAR WAVEGUIDES

Up to now we have discussed the arrays of weakly coupled waveguides with cubic nonlinearity. However, in the last few years a growing interest has been observed in the study of nonlinear optical effects based on the so-called quadratic nonlinearities. In contrast to the conventional studies of quadratic nonlinearities, where the main attention is centered primarily on parametric processes and the frequency conversion, more recent works are focused on the phase modulation of the fundamental as well as the second harmonic waves [25]. This phase modulation accompanies the familiar amplitude modulation, being the basis of any frequency conversion, and it may produce the effects which resemble those known to occur in cubic nonlinear materials. Typical examples are all-optical switching phenomena in interferometric or coupler configurations, and the formation of spatial and temporal solitons in planar waveguides (see, e.g., Ref. [25] and the references therein).

Recently, it was demonstrated theoretically [26–28] that arrays of quadratic nonlinear waveguides represent a convenient system to verify experimentally many theoretical predictions for the dynamics of nonlinear lattices with cubic nonlinearity. The first experimental observation of discrete quadratic solitons has been reported recently by Iwanow and co-workers [19], who demonstrated the formation of discrete quadratic solitons in periodically poled lithium niobate waveguide arrays, excited with fundamental wave pulses at a wavelength of 1572 nm. These experimental observations open many perspectives for employing much larger nonlinearities provided by nonlinear quadratic materials. In this section, we extend the concept of the controlled digitized soliton switching discussed above to the case of quadratic discrete solitons.

A. Discrete model

The standard discrete model for an array of weakly coupled quadratic nonlinear waveguides has the form [9]

$$i\frac{da_n}{dz} + V_a(a_{n+1} + a_{n-1}) + 2\gamma_2 b_n a_n^* = 0,$$

$$i\frac{db_n}{dz} + V_b(b_{n+1} + b_{n-1}) + \beta b_n + \gamma_2 a_n^2 = 0, \qquad (14)$$

where a_n and b_n represent the amplitudes for the fundamental (ω) and second harmonic (2ω) fields in the nth guide and V_a and V_b stand for the linear couplings between the nearest-neighbor waveguides. Parameter γ_2 describes the nonlinear second-order coefficient proportional to the second-order dielectric susceptibility, and β is the effective mismatch between the fields in the array.

As in the case of the cubic nonlinearity, the system (14) possesses two conserved quantities, the total power,

$$P = \sum_n (|a_n(z)|^2 + 2|b_n(z)|^2) \quad (15)$$

and the system Hamiltonian,

$$H = -\sum_n (V_a a_n^* a_{n+1} + V_b b_n^* b_{n+1} + (\beta/2)|b_n|^2 + \gamma_2 a_n^2 b_n^* + \text{c.c.}). \quad (16)$$

However, unlike the case of the cubic nonlinear waveguide arrays, where it is possible to find analytical solutions in the continuum limit which can be used as input profiles for numerical simulations of discrete solitons, in the case of the quadratic nonlinearities no exact solutions are available. Thus, we should resort to the limit of strongly localized modes (SLMs) in order to calculate the PN barrier and use the SLM profile as an input beam profile for the numerical computation of the soliton switching.

B. Localized modes and the PN barrier

As in the case of the cubic nonlinearity, we calculate the PN barrier as a difference between the values of the Hamiltonian for the odd and even strongly localized two-component modes.

Odd modes: We search for approximate solutions of Eq. (14) of the form

$$a_n = a_0\{\ldots,0,a_1 e^{ik},1,a_1 e^{-ik},0,\ldots\}e^{i\lambda_1 z},$$
$$b_n = b_0\{\ldots,0,b_1 e^{2ik},1,b_1 e^{-2ik},0,\ldots\}e^{2i\lambda_1 z}, \quad (17)$$

where a_0 and b_0 are the amplitudes of two harmonics composing a localized mode, k is the initial beam angle or effective kick, λ_1 is the longitudinal propagation constant, and a_1 and b_1 are small parameters that should be determined from the equations. After substituting the ansatz (17) into Eqs. (14) and keeping only linear terms in a_1 and b_1, we obtain $\lambda_1 = 2a_1 V_a \cos(k) + 2\gamma_2 b_0$, $a_0^2 = (b_0/\gamma_2)[2\lambda_1 - \beta - 2b_1 V_b \cos(2k)]$, $a_1 = (V_a/\lambda_1)\cos(k)$, and $b1 = Vb\cos(2k)/(2\lambda 1 - \beta)$. From these relations, we find $\lambda_1 \approx 2\gamma_2 b_0$, which implies

$$a_0^2 \approx 4b_0^2 - (\beta/\gamma_2)b_0,$$
$$a_1 \approx \frac{V_a\cos(k)}{2\gamma_2 b_0} \ll 1, \quad b_1 \approx \frac{V_b\cos(2k)}{4\gamma_2 b_0 - \beta} \ll 1. \quad (18)$$

Even modes. Now we search for approximate solutions of Eqs. (14) of the form

$$\tilde{a}_n = \tilde{a}_0\{\ldots,0,\tilde{a}_1 e^{ik},1,e^{-ik},\tilde{a}_1 e^{-2ik},0,\ldots\}e^{i\lambda_2 z},$$
$$\tilde{b}_n = \tilde{b}_0\{\ldots,0,\tilde{b}_1 e^{2ik},1,e^{-2ik},\tilde{b}_1 e^{-4ik},0,\ldots\}e^{2i\lambda_2 z}, \quad (19)$$

where $\tilde{a}_0$ and $\tilde{b}_0$ are the amplitudes of the coupled harmonics, k is the initial beam angle or effective kick, λ_2 is the longitudinal propagation constant and $\tilde{a}_1$ and $\tilde{b}_1$ are small parameters determined from the equations of motion. After substituting Eq. (19) into Eq. (14) and keeping only linear terms in $\tilde{a}_1$ and $\tilde{b}_1$, we obtain $\lambda_2 = (1+\tilde{a}_1)V_a\cos(k) + 2\gamma_2\tilde{b}_0$, $\tilde{a}_0^2 = (\tilde{b}_0/\lambda_2)(2\lambda_2 - \beta - (1+\tilde{b}_1)V_b\cos(2k))$, $\tilde{a}_1 = (V_a/\lambda_2)\cos(k)$ and $\tilde{b}_1 = V_b\cos(2k)/(2\lambda_2 - \beta)$. From these relations, we find $\lambda_2 \approx V_a\cos(k) + 2\gamma_2\tilde{b}_0$, which implies

$$\tilde{a}_0^2 \approx 4\tilde{b}_0^2 - (\tilde{b}_0/\gamma_2)[\beta - 2V_a\cos(k) + V_b\cos(2k)],$$
$$\tilde{a}_1 \approx \frac{V_a\cos(k)}{V_a\cos(k) + 2\gamma_2\tilde{b}_0} \ll 1, \quad (20)$$
$$\tilde{b}_1 \approx \frac{V_b\cos(2k)}{2V_a\cos(k) + 4\gamma_2\tilde{b}_0 - \beta} \ll 1.$$

From Eq. (18) and Eq. (20), it is easy to see that, in order to obtain SLM, the nonlinear term $\gamma_2 b_0(\tilde{b}_0)$ should be much larger than the linear coupling terms, $V_a\cos(k)$ and $V_b\cos(2k)$.

From Eq. (15) we calculate the total power

$$P_{\text{odd}} \approx a_0^2 + 2b_0^2 + O(a_1^2, b_1^2), \quad (21)$$

$$P_{\text{even}} \approx 2\tilde{a}_0^2 + 4\tilde{b}_0^2 + O(\tilde{a}_1^2, \tilde{b}_1^2), \quad (22)$$

and the Hamiltonian of each mode,

$$H_{\text{odd}} \approx -4a_0^2 a_1 V_a\cos(k) - 4b_0^2 b_1 V_b\cos(2k) - \beta b_0^2 - 2\gamma_2 a_0^2 b_0 + O(a_1^2, b_1^2), \quad (23)$$

$$H_{\text{even}} \approx -2\tilde{a}_0^2(1+2\tilde{a}_1)V_a\cos(k) - 2\gamma_2\tilde{a}_0^2\tilde{b}_0 - 4\tilde{b}_0^2(1 + 2\tilde{b}_1)V_b\cos(2k) - 2\beta\tilde{b}_0^2 + O(\tilde{a}_1^2, \tilde{b}_1^2). \quad (24)$$

We follow the same reasoning as in the case of the cubic nonlinear waveguide arrays and calculate the effective PN barrier. Such calculations look simpler for the physically important case of vanishing mismatch, $\beta \approx 0$. With that assumption, and imposing that the power content of both, odd and even modes, are equal, $P_{\text{odd}} = P_{\text{even}}$, we obtain $3b_0^2 \approx 6\tilde{b}_0^2 + (\tilde{b}_0/\gamma_2)[2V_a\cos(k) - V_b\cos(2k)]$, and then

$$\tilde{b}_0 \approx \frac{b_0}{\sqrt{2}} - \frac{[2V_a\cos(k) - V_b\cos(2k)]}{12\gamma_2}. \quad (25)$$

In terms of b_0, the Hamiltonian of both the modes can be approximated as

$$H_{\text{odd}} \approx -8\gamma_2 b_0^3 + O(a_1^2, b_1^2), \quad (26)$$

$$H_{\text{even}} \approx -4\sqrt{2}\gamma_2 b_0^3 - 8V_a b_0^2\cos(k) - 4V_b b_0^2\cos(2k) + O(a_1\tilde{a}_1, b_1\tilde{b}_1). \quad (27)$$

Finally, in this approximation, we calculate the PN barrier of the strongly localized modes,

$$\Delta^{(2)} = H_{\text{odd}} - H_{\text{even}} \approx -8C\gamma_2 b_0^3 + b_0^2[4V_a\cos(k) + V_b\cos(2k)], \quad (28)$$

where $C = (1 - \sqrt{2}/2)$. This PN barrier for an array of nonlinear quadratic waveguides (28) shows some interesting fea-

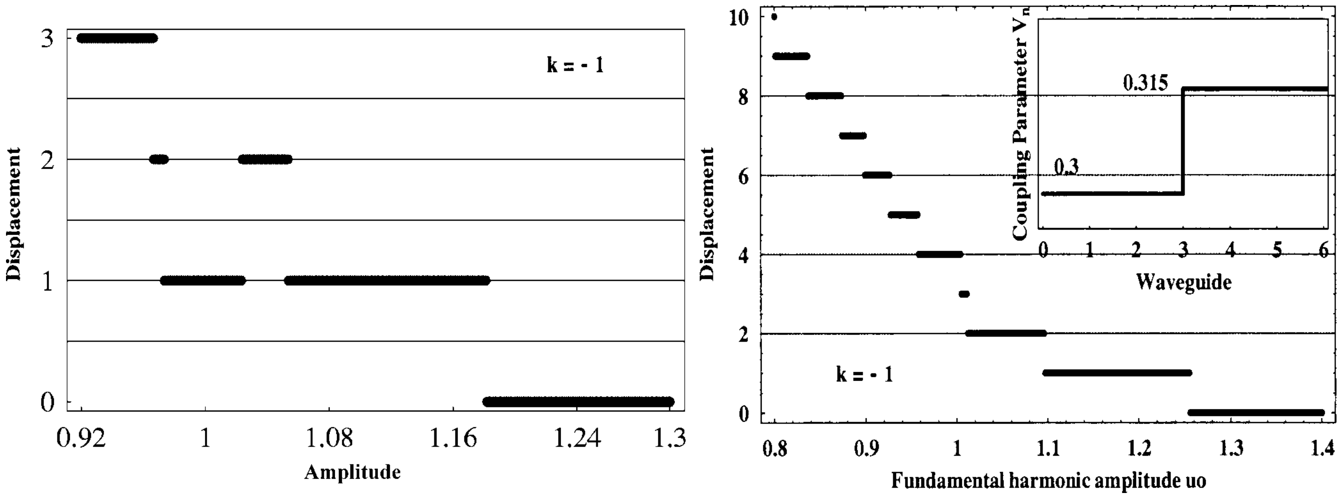

FIG. 6. Digitized switching of a discrete soliton in a quadratic nonlinear waveguide array by varying the fundamental beam intensity, for $k=-1$, $\beta=0$ and $\gamma_2=1$. Left: homogeneous array for $V_a=V_b=0.3$. Right: engineered array for the coupling modulation shown in the inset.

tures: The main term in (28) is cubic in the mode amplitude, while for the cubic case it is quartic [see Eq. (11)]. Also we notice that the first correction to the PN barrier (28) is linear in the couplings, and it depends on the square of the SLM amplitude. This is exactly the same term as in the case of the nonlinear cubic array. Thus, the first-order correction is more important in the nonlinear quadratic array than that in the nonlinear cubic array suggesting that the appropriate engineering of the couplings and/or input kick to achieve digitized switching should be easier to achieve.

For the numerical simulations, we use the initial input in the form of an odd mode,

$$a_n(0) = a_0 a_1^{|n-n_c|} e^{-i(n-n_c)k},$$
$$b_n(0) = b_0 b_1^{|n-n_c|} e^{-2i(n-n_c)k}, \tag{29}$$

for $n-n_c=0, \pm 1$, and $a_n(0)=b_n(0)=0$, otherwise. In Eq. (29) we use $a_0 \approx \sqrt{4b_0^2-(\beta/\gamma_2)b_0}$, $a_1 \approx V_a/2\gamma_2 b_0$ and $b_1 \approx V_b/(4\gamma_2 b_0-\beta)$. We consider an array of 41 waveguides with the initial input centered at the middle, $n_c=20$. For simplicity, we also assume the case of complete phase matching, i.e., $\beta\approx 0$, and identical coupling for both the harmonic fields, $V_{a,n}=V_{b,n}$. Figure 6 shows an uncontrollable switching for a homogeneous array (left-hand side) and a controlled digitized switching of an engineered array (right-hand side) of the discrete two-frequency (fundamental +second harmonic) soliton, by varying the intensity of the input fundamental mode for a fixed parameter $k=-1$. The inset of Fig. 6 (right-hand side) shows the coupling modulation required to achieve this type of engineered soliton switching, which is particularly simple and consists of only a single change of about 5% in the value of the coupling parameter. Figure 7 demonstrates the switching of the discrete mode, composed of the fundamental and second-harmonic fields, to six, five, and four neighboring waveguides, as the intensity of the input fundamental mode is increased. In this respect, it is interesting to point out that in all cases of the digital switching both the fundamental and second-harmonic fields act as a strongly coupled state, and that no lagging behind was observed in any of the modes with respect to the other.

We have performed other simulations with the quadratic nonlinear array including the cases $V_b=0$ (decoupled second-harmonic fields in the array) and $V_b=\alpha V_a$ (reduced coupling of the second-harmonic fields) with $\alpha<1$, etc. In all of those cases, we have observed the digitized switching of the discrete solitons by engineering the coupling in the array as discussed above.

IV. CONCLUSIONS

We have suggested and demonstrated numerically a simple but yet effective method for controlling nonlinear switching of discrete solitons in arrays of weakly coupled optical waveguides. We have demonstrated how to achieve the digitized switching of discrete optical solitons in weakly coupled arrays of cubic and quadratic nonlinear waveguides described, in the framework of the tight-binding approximation, by discrete models such as the DNLS equation with a steplike variation of the waveguide coupling parameter. Our approach involves a weak steplike modulation of the coupling strength (or, equivalently, distance between the waveguides) in the arrays with the period larger than the waveguide spacing. Such superlattice waveguide structure

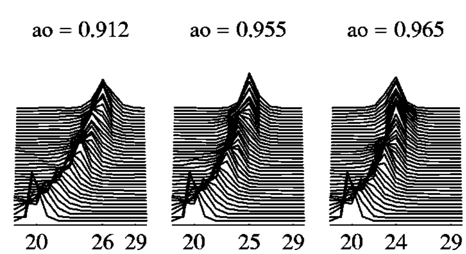

FIG. 7. Switching to 6, 5, and 4 sites of a discrete quadratic soliton (SLM) with a slight intensity variation of the fundamental mode, with the coupling modulation shown in the inset of Fig. 6.

permits the modification of the trapping properties of the array by fine-tuning the strength of the effective Peierls-Nabarro (PN) potential. In particular, we have estimated the PN potential analytically for the quadratic nonlinear array for the first time, to our knowledge. We have demonstrated the digitized switching of a narrow input beam for up to 11 waveguides, in the case of the cubic nonlinear array, and up to 10 waveguides, in the case of quadratic nonlinear array.

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